Cake Filtration

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<u>Cake Filtration</u>

- **Cake filtration consists of passing a solid suspension (slurry) through a porous medium or septum (e.g., a woven wire). The solids in the slurry are retained on the surface of the medium where they build up, forming an increasing thicker *cake*.**
- **As more slurry is filtered the solids retained on the medium provide most of filtering action. <u>In cake filtration the cake is the real filtering element.</u>**

Cake Filtration (continued)

- **As time goes by the thickness of the cake increases, as more solids are filtered. This results in a corresponding increase of the pressure resistance across the cake.**
- **If the cake is incompressible (i.e., it does not change its volume as pressure builds up) the pressure resistance increases proportionally to the cake thickness.**
- **However, since most cakes are compressible the pressure across the cake typically increases even faster than the cake build-up.**

Cake Filtration (continued)

- **The cake is removed intermittently during batch filtration processes. This is done by taking the filter off line and manually or automatically collecting the cake.**
- **The cake is removed continuously in continuous processes, for example by scraping the cake with blades, as in rotating filters.**
- **Cake washing and drying operations can also be incorporated in the operation of most filters.**

Examples of Cake-Forming Filters

- Filter presses
- Belt filters
- Vacuum filters:
  - Rotary vacuum belt filters
  - Rotary vacuum precoat filters
  - Vacuum disk filters

Example of a Filter Press

After Metcalf and Eddy, *Wastewater Engineering*, 1991, p. 869

Cross Section of a Filter Press

After Freeman, *Standard Handbook of Hazardous Waste Treatment and Disposal*, 1989, p. 7.9

<u>Example of a Belt Filter</u>

After Freeman, *Standard Handbook of Hazardous Waste Treatment and Disposal*, 1989, p. 7.10

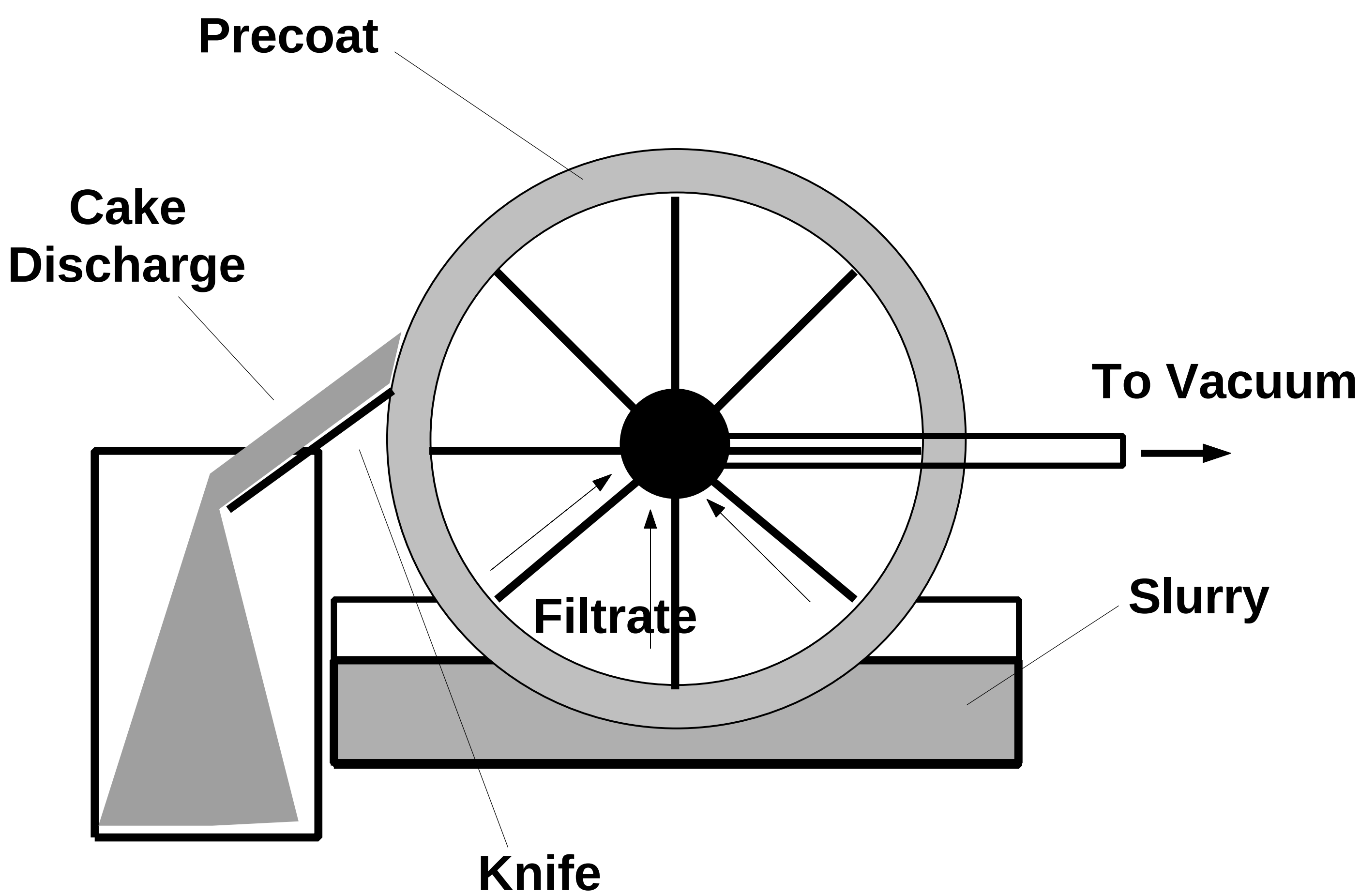
Schematic of a Belt Press Filter

After Vesilind, *Treatment and Disposal of Wastewater Sludges,* 1979, p.156.

<u>Example of a Rotary Vacuum Belt Filter</u>

After Freeman, *Standard Handbook of Hazardous Waste Treatment and Disposal*, 1989, p. 7.8

Example of a Rotary Vacuum Precoat Filter



Precoats and Filter Aids

- **A *precoat* is a layer of fine particulate material (e.g., perlite) added on to the filter septum before filtration to form a coating cake**
- **During filtration the filtered solids in the slurry may clog the filter and reduce the rate of filtration. This happens especially if the resulting cake is very compressible**
- **In such cases a *filter aid* made of fine particles of a hard but porous material (such as perlite) having good filtering properties is added to the slurry to prevent and filtered with the slurry solids**

Common Types of Precoats and Filter Aids

- ***Diatomaceous earth (diatomite)***

  A light siliceous material derived primarily from sedimented diatoms (minute planktonic unicellular or colonial algae with silicified skeletons). Typical bulk density: 0.32 g/cm^3

- ***Perlite***

  A volcanic glass made of siliceous rock having a concentric shelly structure. Typical bulk density: 0.16 g/cm^3

Analysis of Cake Filtration

Analysis of Suspended Solids Removal During Cake Filtration

- **As the suspension moves through the filter medium (septum) the suspended solids are stopped by the filter septum forming a filter cake on top of the filter septum**
- **As more solids suspension passes through the filter the cake builds up providing most of the filtering action for the incoming suspension**
- **Equations can be written to describe the removal of the particles in suspension by the filter (i.e., the formation of the cake) and the pressure drop of the fluid as it passes through the cake**

Important Variables in Cake Filtration

- **Available pressure drop across cake, DP (Pa)**
- **Area of filtration, A (m^2)**
- **Specific resistances of cake, a (m/kg)**
- **Specific resistances of medium (septum), R_m (1/m)**
- **Fluid superficial velocity, u_s (m/s)**
- **Size of cake particles, D_p (m)**
- **Shape factor for particles, f_s**
- **Type of solids in suspension**
- **Cake void fraction, e (void volume/total bed volume)**
- **Time, t (s)**

Important Variables in Cake Filtration

- **Cake thickness, L (m)**
- **Concentration of solids in wastewater, X_w (g/L)**
- **Residual concentration of solids in filtrate, X_F (g/L)**
- **Mass fraction of solids in cake, X_C' (g/g)**
- **Cumulative volume of wastewater fed to filter, V_W (L)**
- **Cumulative volume of filtrate generated, V_F (L)**
- **Cumulative mass of wet cake, m_c (g)**
- **Mass of solids in the cake per volume of filtrate, X_S (g/L)**
- **Density of wastewater, r_w (g/L)**
- **Density of filtrate, r_f (g/L)**

Approach to Cake Filtration as a Batch Process

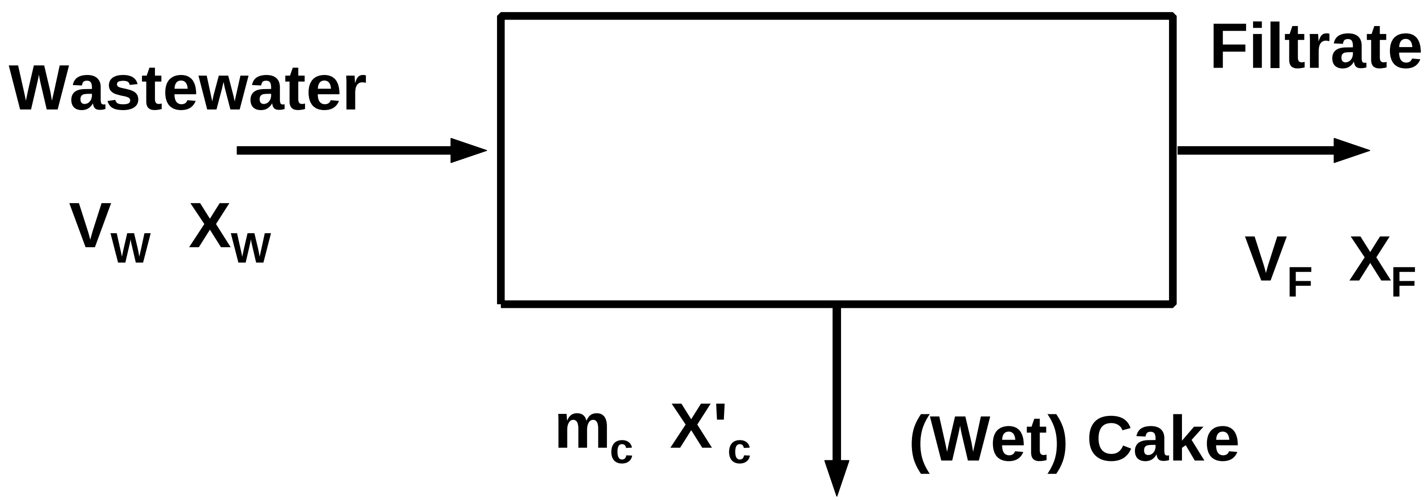
- **Cake filtration is intrinsically a batch process. Hence, it can be expected that as filtration proceeds the cake will build up and the pressure drop across the cake will increase.**
- **Mathematical modeling of batch cake filtration is based on the determination of the rate of formation of the cake and the calculation of pressure drop at any given time.**

Approach to Cake Filtration as a Batch Process (continued)

- **Integral quantities (such as the cumulative volume of filtrate produced during a time interval, or the mass of the cake generated during the same interval) can be calculated by integration of the basic instantaneous mass balances. In these equations the <u>pressure drop is typically a function of time</u>.**
- **<u>Continuous filtration</u> is often modeled as a succession of batch processes carried out over infinitesimally small time intervals.**

Mass Balance Around a Filter

For a filter operating in a batch mode the following diagram can be drawn:



Definition of X_s

X_s is defined as the mass of (dry) solids in the cake per volume of filtrate generated.

From this definition it is that:

$$X_s = \frac{\text{mass of solids in cake}}{\text{volume of filtrate}} = \frac{X_c' m_c}{V_F}$$

Relationship Between Solid Concentrations Around a Filter

Mass balances around the filter give:

$$X_w V_W = X_c' m_c + X_F V_F \qquad \text{(solids)}$$

$$r_w V_W = m_c + r_F V_F \qquad \text{(overall)}$$

Relationship Between Solid Concentrations Around a Filter (cont.'d)

After an explicit expression for V_F has been obtained it can be substituted in the equation defining X_s to get:

$$X_s = X_c' \cdot \frac{r_W X_F - r_F X_W}{X_W - r_W X_c'}$$

If the densities of the wastewater and the filtrate are the same, then:

$$r_W = r_F = r$$

$$X_s = r X_c' \cdot \frac{X_F - X_W}{X_W - r X_c'}$$

Relationship Between Solid Concentrations Around a Filter (cont.'d)

Special case: A common situation is that in which all the solids contained in the suspension are removed by the filter and contribute to the formation of the cake. In other words, the filtrate does not contain any solids. In such a case it is:

$$X_F \equiv 0$$

and the expression for X_s becomes:

$$X_s = r\,X_c' \cdot \frac{X_W}{r\,X_c' - X_W}$$

Relationship Between Solid Concentrations Around a Filter (cont.'d)

Note that, in general, X_s is different from X_W

Only if:

$$r\,X_c' \gg X_W$$

it would then be that:

$$X_s = r\,X_c' \cdot \frac{X_W}{r\,X_c' - X_W} \cong r\,X_c' \cdot \frac{X_W}{r\,X_c'} \cong X_W$$

Cumulative Mass Balance for the Solids in the Cake

At a generic time *t* a cumulative mass balance for the solids in the cake (i.e., the solids that have contributed to the formation of the cake in the time interval 0-*t*) gives:

$$\left\{\begin{matrix}\text{solids accumulated in the cake} \\ \text{during time t}\end{matrix}\right\} =$$

$$\left\{\begin{matrix}\text{solids removed from suspension} \\ \text{during time t}\end{matrix}\right\}$$

Cumulative Mass Balance for the Solids in the Cake

The previous equation can be rewritten symbolically, for a generic time *t*, as:

$$L\,A\left(1-e\right)r_s + X_W\left(e\,L\,A\right) = X_s V_F$$

The first term represents the *mass of solids in the solid component of the cake* at time *t*; the second term is the *amount of solids still in suspension in the water contained in the cake*; and the third term is the *amount of solids removed from the filtrate* (and now held in the cake).

Remark: *L*, e, and V_F can all be functions of time.

Cumulative Mass Balance for the Solids in the Cake (continued)

The volume of water contained in the cake (εLA) is typically much smaller than the volume of filtrate, V_F, produced during the time interval 0-*t*. Furthermore, X_s and X_w are of the same order of magnitude. Then, one can safely assume that:

$$X_s V_F >> X_W \left(\varepsilon L A \right)$$

Hence, the cumulative mass balance for the solids in the cake becomes:

$$L A \left(1 - \varepsilon \right) \rho_s \cong X_s V_F$$

Cake Thickness, *L*, as a Function of Volume of Liquid Passed Through the Filter

The previously derived mass balance equation for the solids in the cake:

$$L\,A\left(1-e\right)r_s = X_s V_F$$

can be rewritten as:

$$\boxed{L = \frac{X_s V_F}{A\left(1-e\right)r_s}}$$

where *L*, e, and V_F can all be functions of time.

Shape Factor of Particles in Cake

The particle shape factor, f_p, is defined as:

$$f_p = \frac{\text{Surface area of sphere having same volume as particle}}{\text{Surface area of particle}}$$

i.e.,

$$f_p = \frac{6\, pD^2_{sph}}{pD^3_{sph}} \cdot \frac{V_p}{A_p} = \frac{6V_p}{D_{sph}\, A_p}$$

where D_{sph} is the diameter of a sphere having the same volume as the particle.

Relationship Between D_p, D_{sph}, and f_p

Since:

$$D_p = \frac{6V_p}{A_p}$$

and:

$$f_p = \frac{1}{D_{sph}} \cdot \frac{6V_p}{A_p}$$

then:

$$D_p = f_p D_{sph}$$

Approximate Relationship Between D_p and Sieve Opening

The assumption is often made that:

$$D_{sph} \approx \overline{D_p}$$

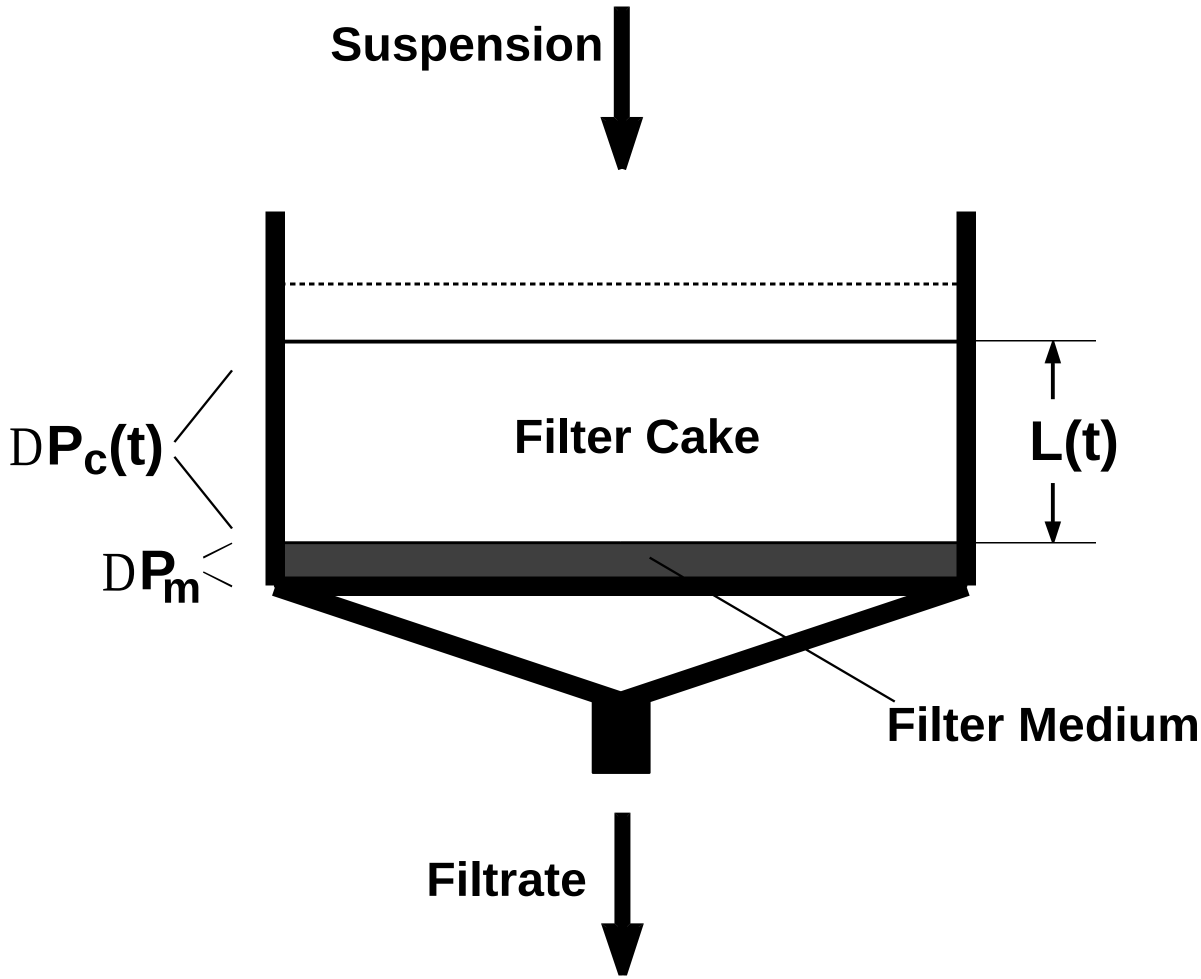
where $\overline{D_p}$ **is the average size of the particles whose size is between two sieve openings**

$$\overline{D}_p = \sqrt{D_{s1} D_{s2}}$$

and where D_{s1} and D_{s2} are the sieve openings. Then:

$$D_p = f_p D_{sph} \cong f_p \overline{D_p}$$

Pressure Drop During Cake Filtration



Pressure Drop During Cake Filtration

At any time, *t*, the pressure drop experienced at that time by a suspension passing through a filter cake supported by a filter medium (or septum) is:

$$\Delta P(t) = \Delta P_c(t) + \Delta P_m$$

where:

$\Delta P(t)$ **= total pressure drop across filter**

$\Delta P_c(t)$ **= pressure drop due to filter cake**

ΔP_m **= pressure drop due to filter medium**

Remark: during batch filtration the cake can be expected to build up, and the pressure drop to increase as time passes.

Pressure Drop Across Filter Cake

Since the liquid passing through the filter cake moves in laminar flow (because of the small pores of the cake and the slow fluid velocity) the Blake-Kozeny equation can be used (instead of the more general Ergun equation) to describe the dependence of the pressure drop through the cake with the superficial velocity, u_s:

$$\Delta P_c = \frac{150}{\mathrm{Re}_p}\left[\frac{(1-e)^2}{e^3}\right]\frac{L}{D_p}\, r_L\, u_s^2$$ **Blake-Kozeny equation**

where DP_c is the pressure drop *through the cake*, and is, in general, a function of time.

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Pressure Drop Across Filter Cakes

Substituting the expression for Re in the Blake-Kozeny equation gives:

$$\Delta P_c = 150\, m \left[\frac{(1 - e)^2}{e^3} \right] \frac{L}{D_p^2} u_s$$

As before, DP_c is typically a <u>function of time</u>, since the cake thickness, *L*, the superficial velocity, u_s, and the void fraction, *e*, can all change with time.

Pressure Drop Across Filter Cakes (Carman-Kozeny Equation)

For filter cakes the constant 150 may not be appropriate since the cake particles are compressible.

Therefore the Blake-Kozeny equation is often rewritten to produce to so-called Carman-Kozeny equation:

$$\Delta P_c = k_1 \, m \left[\frac{(1-e)^2}{e^3} \right] \frac{L}{D_p^2} u_s \quad \textbf{Carman-Kozeny equation}$$

where: k_1 = proportionality constant.

Superficial Velocity in Cake Filtration

As before, the superficial (or approach) velocity is defined as the velocity of the liquid as it flows through a cross section equal to that of the tank (or filter vessel) in the absence of the cake. It is also equal to the filtrate flow rate, Q_F, divided by the total cross-sectional area normal to flow, i.e.:

$$u_s = \frac{Q_F}{A} = \frac{dV_F}{dt}\frac{1}{A}$$

where:

A = cross sectional area or empty filter vessel

V_F = volume of filtrate passed through the cake during time t

Equation for Pressure Drop in the Cake

Combining together the equations:

$$\Delta P_c = k_1 m \left[\frac{(1-e)^2}{e^3} \right] \frac{L}{D_p^2} u_s \qquad u_s = \frac{Q_F}{A} = \frac{dV_F}{dt} \frac{1}{A}$$

and:

$$L = \frac{X_s V_F}{A(1-e)\, r_s}$$

the following expression for DP_c is found:

$$\Delta P_c = \left\{ \frac{k_1}{r_s} \left[\frac{(1-e)^2}{e^3} \right] \frac{1}{(1-e) D_p^2} \right\} m \frac{X_s V_F}{A^2} \frac{dV_F}{dt}$$

Equation for Pressure Drop in the Cake

The previous equation can be re-arranged to give the final equation for the pressure drop in the cake:

$$\Delta P_c = a\, m \frac{X_s V_F}{A^2} \frac{dV_F}{dt}$$

where a **=** ***specific*** **cake resistance to filtration, is given by:**

$$a = \frac{k_1}{r_s D_p^2} \frac{1 - e}{e^3}$$

DP_C**,** L**,** a**, and** V_F **can all be functions of time.**

Equation for Pressure Drop in Filter Medium (Septum)

The pressure drop across the filter medium (septum) can also be expressed using the Carman-Kozeny equation that can be rewritten as:

$$\Delta P_m = k_2\, m \left[\frac{\left(1 - e_m\right)^2}{e_m^3} \right] \frac{L_m}{D_{pm}^2} u_s$$

where the subscript "m" refers to the medium and the superficial velocity is given by:

$$u_s = \frac{Q_F}{A} = \frac{dV_F}{dt} \frac{1}{A}$$

<u>Equation for Pressure Drop in Filter Medium (continued)</u>

The resulting expression of the <u>pressure drop in the medium</u> is:

$$\Delta P_m = \mu R_m \frac{1}{A}\frac{dV_F}{dt}$$

where:

$$R_m = k_2\left[\frac{\left(1-\varepsilon_m\right)^2}{\varepsilon_m^3}\right]\frac{L_m}{D_{pm}^2}$$

with R_m = *specific* resistance of medium to filtration

<u>Equation for Total Pressure Drop During Cake Filtration</u>

Recalling that the total pressure drop in a filter is:

$$\Delta P(t) = \Delta P_c(t) + \Delta P_m$$

it is:

$$\Delta P(t) = \left[a(t)\, m \frac{X_s V_F(t)}{A^2} + m R_m \frac{1}{A} \right] \frac{dV_F}{dt}$$

Since by definition it is: $Q_F(t) = dV_F/dt$, then:

$$\boxed{\frac{dV_F}{dt} = Q_F(t) = \frac{A^2\, \Delta P(t)}{m\left[a(t)\, X_s V_F(t) + A R_m\right]}}$$

<u>This is the main design equation for cake filters</u>.

<u>Specific Cake Resistance and Cake Compressibility</u>

From the expression for a:

$$a = \frac{k_1}{r_s D_p^2} \frac{1-e}{e^3}$$

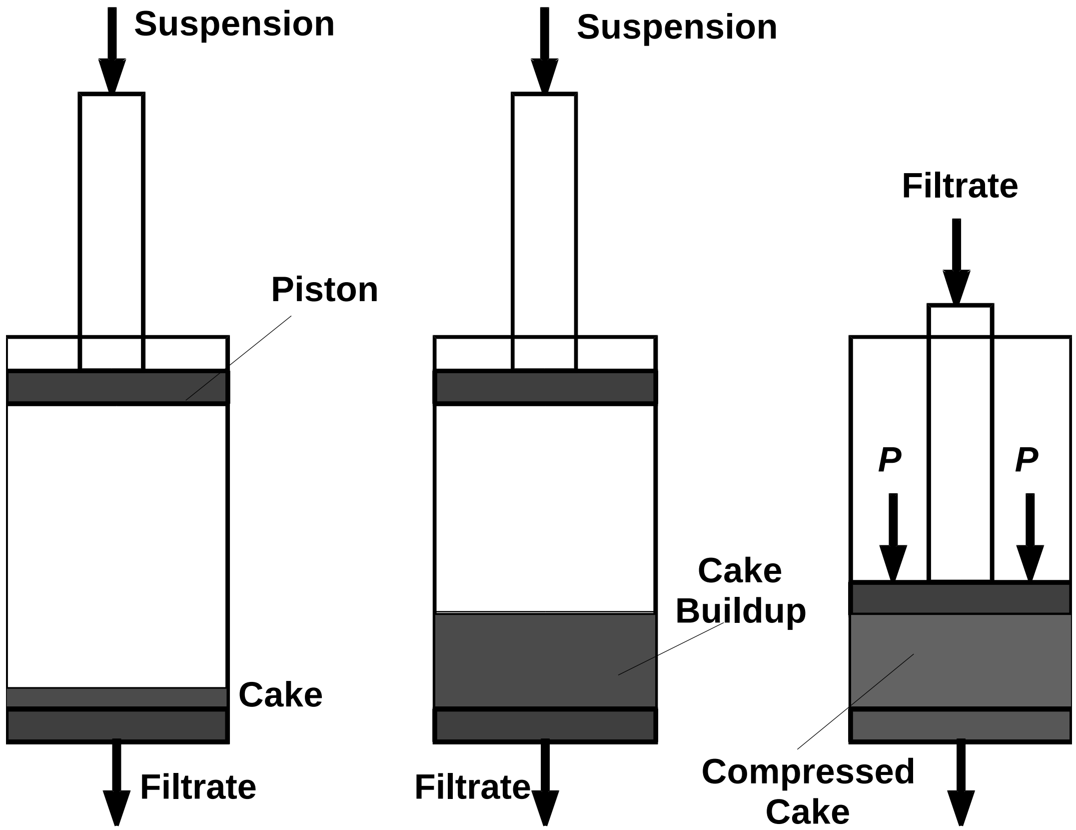
one can *incorrectly* assume that the pressure across the cake has no impact on specific cake resistance. In fact, the void fraction e for most cakes can be significantly affected by pressure, since the cake is often <u>compressible</u>. Since the pressure drop changes with time the void fraction e can also be a function of time, at least in principle.

Specific Cake Resistance and Cake Compressibility

In practice, it is convenient to carry out experiments to determine:

- **the specific cake resistance under no pressure difference (no compression). Cake is built up by gravity filtering;**
- **the effect of pressure difference across the cake on the specific cake resistance. Cake is built up first and then compressed to a know pressure with a piston provided with a porous bottom. Filtrate is passed trough the cake.**

Specific Cake Resistance and Cake Compressibility



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<u>Specific Cake Resistance and Cake Compressibility (continued)</u>

Possible results of cake compression experiment:

- **cake is <u>incompressible</u>. Cake resistance, a is independent of D*P;***
- **cake is <u>compressible</u>. Cake resistance is expressed as:**

$$a = a_o (\Delta P)^s$$

with: $a_{\mathbf{o}}$ = empirical constant

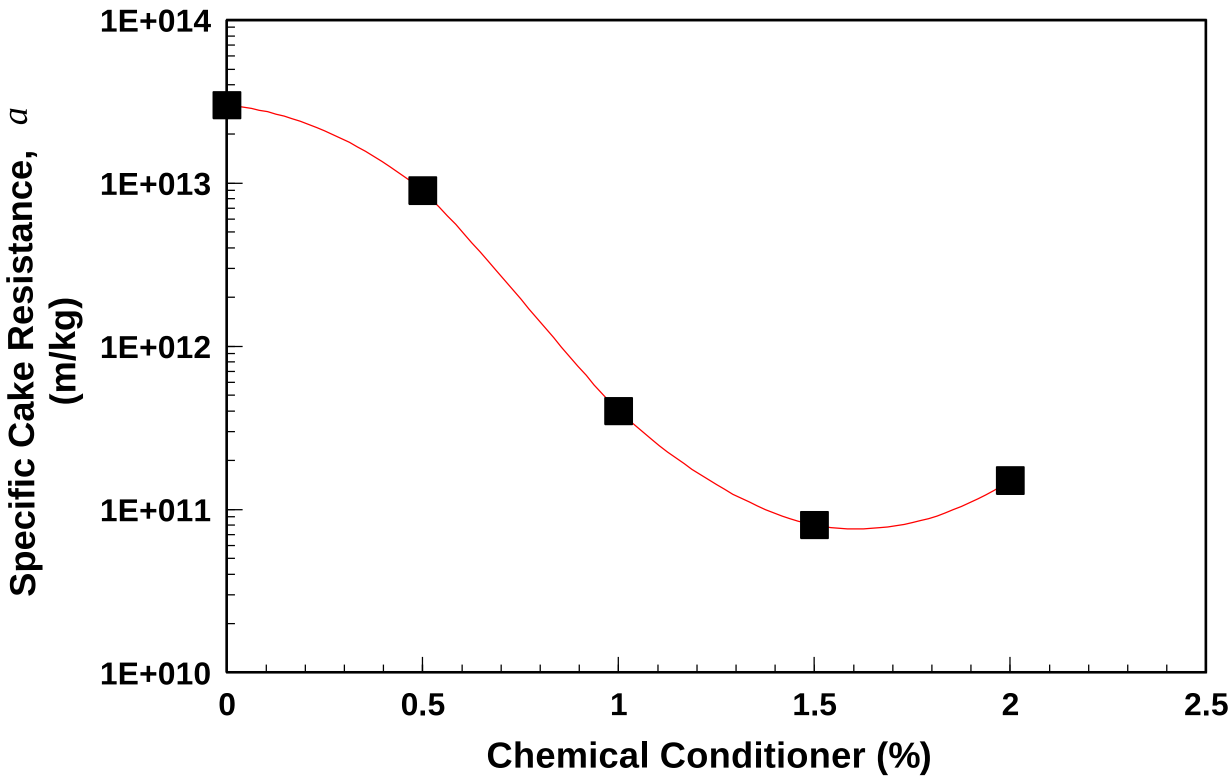
***s = coefficient of compressibility* (typical range for most domestic sludges: 0.4-0.9; lime sludges: 1.05; sand: 0).**

Specific Cake Resistance

Typical values of the specific cake resistance, a, are in the following ranges:

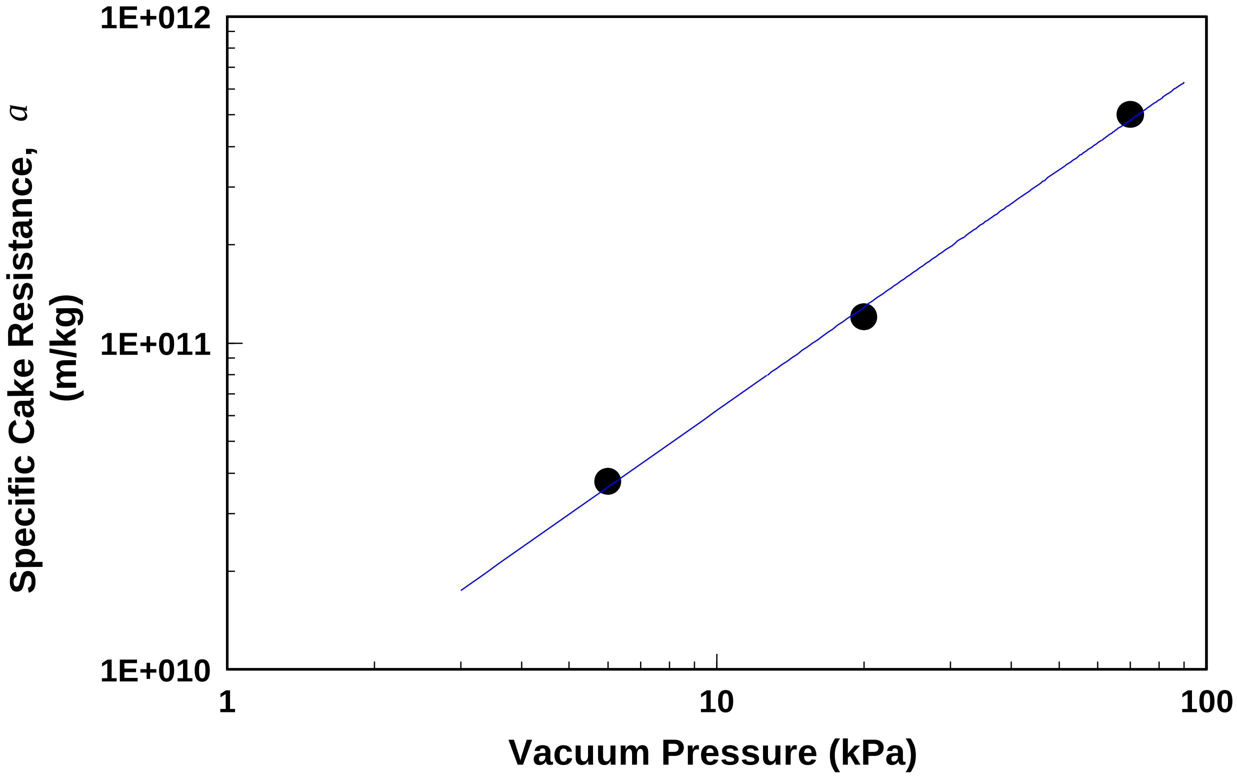
- **10^{13}-10^{15} m/kg for raw sludges;**
- **10^{11}-10^{12} m/kg for well conditioned sludges.**

Typical Specific Cake Resistance with Chemical Conditioning



After Vesilind, *Treatment and Disposal of Wastewater Sludges,* 1979, p.156.

Compressibility of Sludges as Measured by the Specific Resistance Test



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Batch Filtration Operations

Batch cake filtration is typically carried out under one of the following conditions:

- **Constant filtrate flow rate. Since the pressure drop across the filter increases as a result of cake buildup this condition implies that the upstream pressure must be increased with time.**
- **Constant pressure drop across the filter. This condition implies that the filtrate flow rate declines as the cake builds up.**
- **Variable flow rate and variable pressure drop.**

Batch Cake Filtration at Constant Filtrate Flow Rate

In some cases cake filtration is carried out using a positive displacement pump. This results in a <u>constant flow rate</u> process. Then:

$$Q_F = \text{constant}$$

Recalling the design equation for cake filters it is:

$$\frac{dV_F}{dt} = Q_F = \frac{A^2 \, \Delta P(t)}{\mu\left[\alpha(t) X_s V_F(t) + A R_m\right]}$$

Important: although $Q_F = dV_F/dt$ is a constant, V_F (the total filtrate at time t) is <u>not</u>. In fact, it is:

$$dV_F = Q\,dt \qquad \Rightarrow \qquad V_F = Q_F\, t$$

Batch Cake Filtration at Constant Filtrate Flow Rate (cont' d)

Assuming that the cake is <u>not</u> compressible (i.e., a is independent of D*P*) the pressure buildup while operating at constant filtrate flow rate (i.e., constant Q_F) is given by:

$$\Delta P(t) = \frac{m\left[a\ X_s V_F(t) + A R_m\right]}{A^2} Q_F$$

i.e., recalling that $V_F = Q_F \cdot t$:

$$\boxed{\Delta P(t) = \frac{m a\ X_s Q_F^2}{A^2} t + \frac{m R_m Q_F}{A}} \qquad \left(\text{for } \ Q_F = \text{constant}\right)$$

Applications of Batch Cake Filtration at Constant Filtrate Flow Rate

- **Batch cake filtration at constant filtrate flow rate is used primarily in sludge dewatering;**
- **The type of filters that utilizes filtration method is the filter press;**
- **Positive displacement pumps are used to force the suspension through the filter;**
- **Gauge pressures up to 225 psi (15 atm) are used.**

Batch Cake Filtration at Constant D*P*

If the pressure across the filter is constant the general filtration equation:

$$\frac{dV_F}{dt} = Q_F(t) = \frac{A^2\,\Delta P(t)}{m\left[a(t)\,X_s V_F(t) + A R_m\right]}$$

becomes:

$$\frac{dV_F}{dt} = Q_F(t) = \frac{A^2\,\Delta P}{m\left[a\;X_s V_F(t) + A R_m\right]}$$

Note that the coefficient <u>*a* is constant (but not necessarily equal to a_o) even if the cake is compressible, since D*P* = constant</u>.

Batch Cake Filtration at Constant D*P* (continued)

Since D*P* = constant, the previous equation can be integrated by separating variables:

$$\int_0^{V_F} \frac{m\left(a\ X_s V_F^{'} + A R_m\right)}{A^2\,\Delta P}\, dV_F^{'} = \int_0^t d\,t^{'}$$

Integration of this equation yields:

$$\boxed{\frac{m\,a\ X_s}{2A^2\,\Delta P} V_F^2(t) + \frac{m R_m}{A\,\Delta P} V_F(t) = t} \qquad (\text{for } \Delta P = \text{constant})$$

Batch Cake Filtration at Constant D*P* (continued)

The previous equation can be rewritten as:

$$h V_F^2(t) + g V_F(t) = t$$

where D*P* is constant, and the parameters h **and** g **are given by the equations:**

$$h = \frac{m a \ X_s}{2 A^2 \Delta P} \qquad \textbf{and} \qquad g = \frac{m R_m}{A \Delta P}$$

Batch Cake Filtration at Constant D*P*: Determination of Filtration Parameters

The determination of h and g from <u>batch experiments conducted at constant D*P*</u> can be made by rearranging the equation:

$$\frac{m a\ X_s}{2A^2\,\Delta P} V_F^2(t) + \frac{m R_m}{A\,\Delta P} V_F(t) = t$$

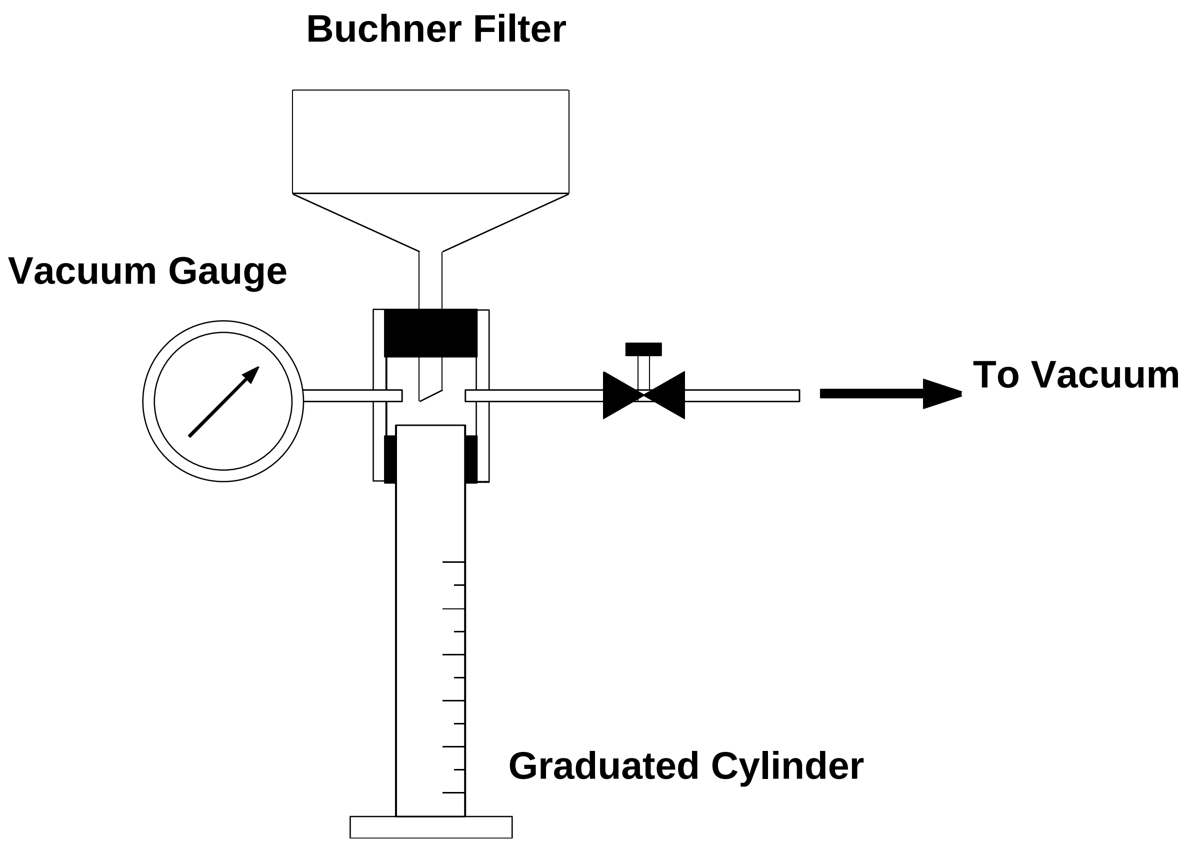
as:

$$\frac{t}{V_F(t)} = \frac{m a\ X_s}{2A^2\,\Delta P} V_F(t) + \frac{m R_m}{A\,\Delta P}$$

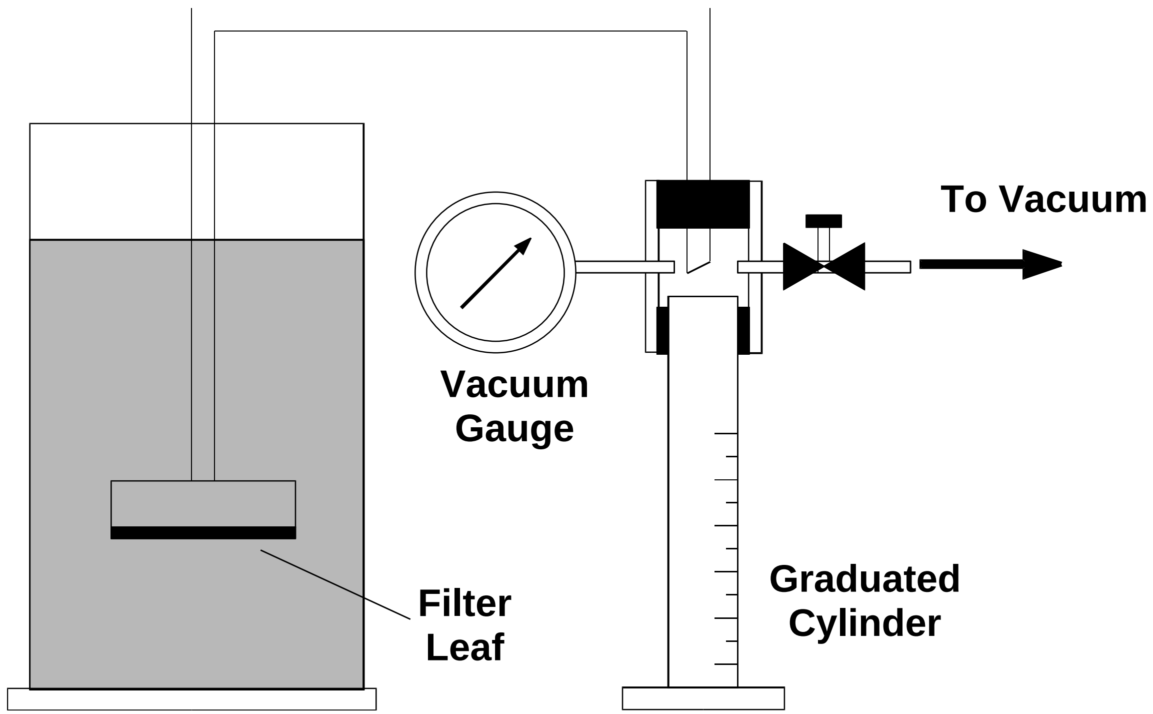
with: $\text{slope} = h = \dfrac{m a\ X_s}{2A^2\,\Delta P}$ and $\text{intercept} = g = \dfrac{m R_m}{A\,\Delta P}$

and *t*/*V*$_F$ = y-coordinate and *V*$_F$ = x-coordinate

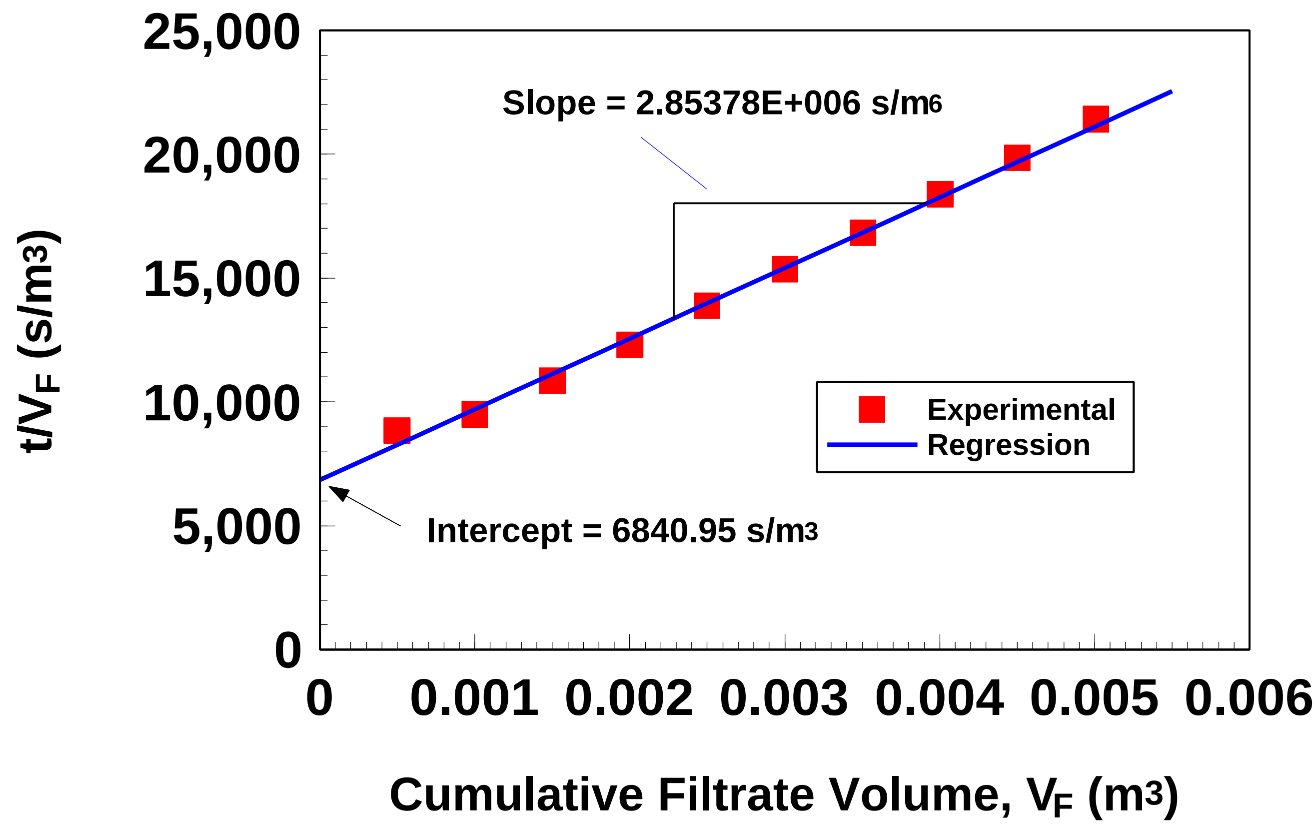
Determination of the Specific Cake Resistance Through Batch Filtration Experiments at Constant ΔP: Buchner Funnel Apparatus



Determination of the Specific Cake Resistance Through Batch Filtration Experiments at Constant ΔP: Filter Leaf Apparatus



Example of Experimental Determination of Filtration Constants



Batch Cake Filtration at Constant ΔP: Approximate Equations

If the resistance of the filter medium, R_m, is very small in comparison to the cake resistance, α, the batch filtration equation can be rewritten as:

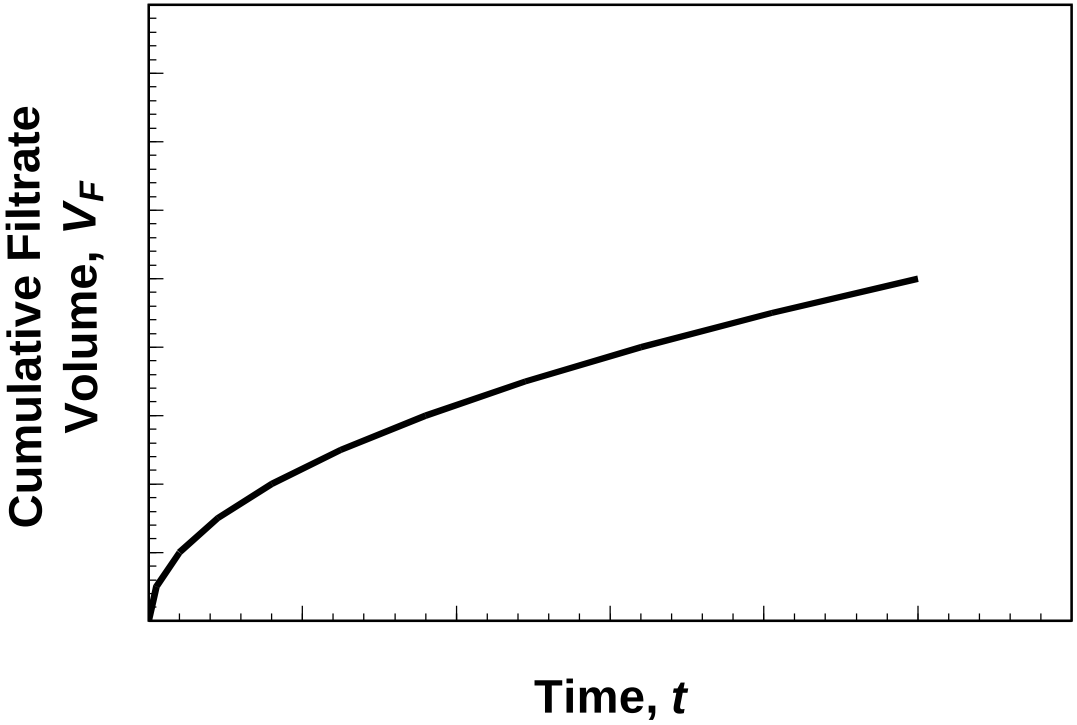
$$t \cong \frac{\mu \alpha X_s}{2 A^2 \Delta P} V_s^2(t) \qquad \Rightarrow \qquad V_F(t) \cong \sqrt{\frac{2 A^2 \Delta P}{\mu \alpha X_F} t} = \sqrt{\frac{t}{\eta}}$$

i.e.:

$$Q_F(t) = \frac{dV_F}{dt} \cong \frac{1}{2}\sqrt{\frac{2 A^2 \Delta P}{\mu \alpha X_F t}} = \frac{1}{2}\sqrt{\frac{1}{\eta t}}$$

Note that $Q_F(t) \rightarrow 0$ for $t \rightarrow \infty$.

Batch Cake Filtration at Constant ΔP: Plot of Approximate Expression for $V_F(t)$



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Applications of Batch Cake Filtration at Constant ΔP

- **Batch cake filtration at constant ΔP is used primarily in sludge dewatering;**
- **The types of filters using this filtration method include:**
  - **Filter presses**
  - **Belt filter presses**

Continuous Cake Filtration at Constant D*P*: Rotary Vacuum Filters

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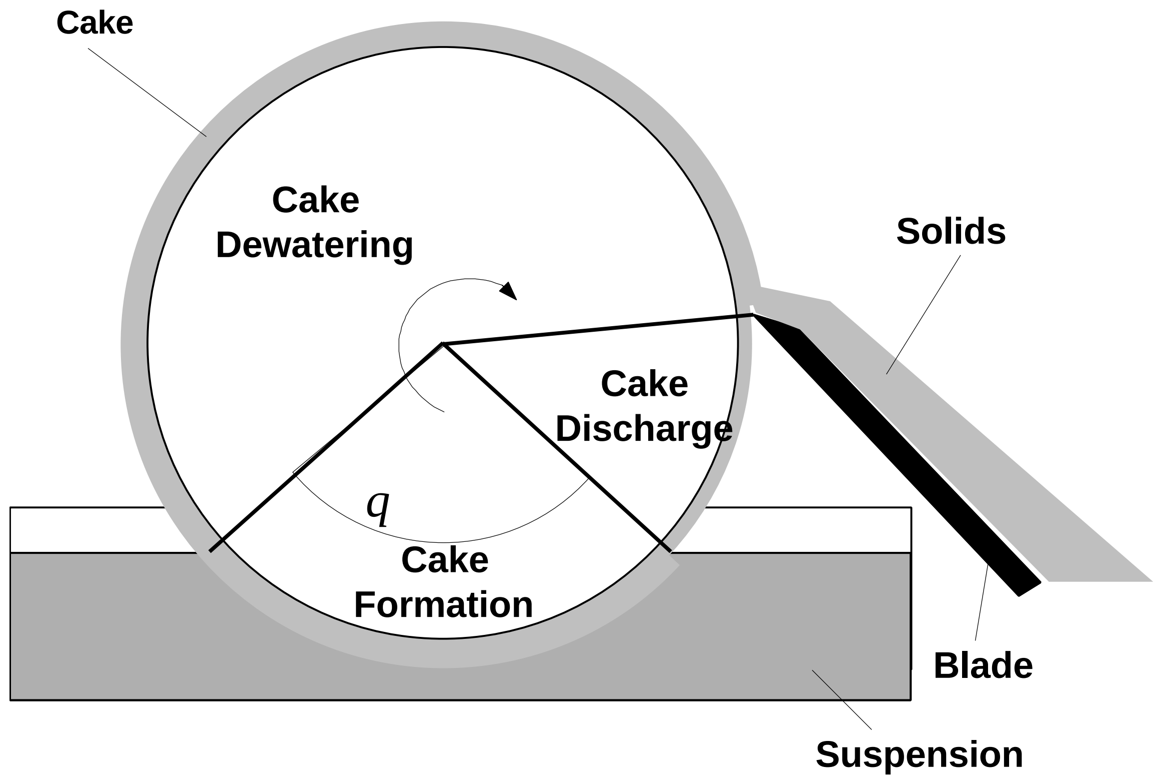
Continuous Cake Filtration at Constant ΔP: Rotary Vacuum Filter



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Continuous Cake Filtration at Constant ΔP: Rotary Vacuum Filters

- **In continuous filtration operations, such as those involving vacuum rotary filters, each filter element undergoes a batch cake filtration, followed by a cake dewatering phase, and a cake discharge phase with each rotation.**
- **The filter cake is formed under a constant ΔP driving force generated by a vacuum.**
- **The filter cake is formed only during the time period when the filter surface is immersed in the suspension.**

Continuous Cake Filtration at Constant D*P*: Cycle Time

The immersion period in each cycle (i.e., for each full rotation of the filter drum) is given by:

$$t = f_k\, t_c = f_k \frac{2\,p}{w} = \frac{q}{2\,p}\frac{2\,p}{w} = \frac{q}{w}$$

where: t_c **= cycle time (time for one full rotation)**

f_k **=** **fraction of cycle time available for cake formation = fraction submergence of drum surface**

q **=** **angle comprising the sector immersed in suspension (rad)**

w **=** **rotational (angular) velocity (rad/s)**

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Continuous Cake Filtration at Constant ΔP: Filtration Equation

During the time period $t = f_k\, t_c$ the filter cake in a rotary filter is formed just as in a batch operation. The (batch) filtration equation for part of the continuous process is:

$$\frac{\mu \alpha X_s}{2 A^2 \Delta P} V_F^2(t) + \frac{\mu R_m}{A \Delta P} V_F(t) = t = f_k\, t_c$$

since ΔP is constant.

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Continuous Cake Filtration at Constant D*P*: Filtrate Generated During a Cycle

The previous quadratic expression is an equation in V_F that can be solved for V_F and rearranged to give:

$$V_F(t) = \frac{A}{a\ X_s}\left[-R_m + \sqrt{R_m^2 + \frac{2 f_k\, t_c\, a\ X_s\ \Delta P}{m}} \right]$$

Continuous Cake Filtration at Constant ΔP: Filtrate Flux

The previous equation can be rearranged to give:

$$\text{Filtrate flux} = \frac{Q_F}{A} =$$

$$= \frac{V_F}{A t_c} = \frac{1}{\alpha X_s}\left[-\frac{R_m}{t_c} + \sqrt{\frac{R_m^2}{t_c^2} + \frac{2 f_k\, \alpha\, X_s\, \Delta P}{\mu t_c}}\right]$$

which predicts the filtrate flux, i.e., the amount of filtrate V_F produced per unit filter area during a cycle (or full rotation) lasting a time interval t_c.

Q_F = average filtrate flow rate during the interval t_c

Continuous Cake Filtration at Constant Δ*P*: Filter Loading

This equation can be rewritten to give the amount of filter solids loading, G_k, produced during the same cycle:

$$G_k = \frac{V_F X_s}{A t_c} = \frac{1}{a}\left[-\frac{R_m}{t_c} + \sqrt{\frac{R_m^2}{t_c^2} + \frac{2 f_k\ a\ X_s\ \Delta P}{m t_c}}\right]$$

where:

G_k = filter solids loading (kg/m² s) = amount of solids filtered per unit filter area over a time interval t_c.

Continuous Cake Filtration at Constant ΔP: Continuous Operation

Since a continuous process is nothing more than a sequence of cycles, each one lasting t_c, then:

$$\text{Filtrate flux} = \frac{Q_F}{A} = \frac{1}{a\,X_s}\left[-\frac{R_m}{t_c} + \sqrt{\frac{R_m^2}{t_c^2} + \frac{2 f_k\ a\ X_s\ \Delta P}{m t_c}}\right]$$

The equation:

$$G_k = \frac{V_F\,X_s}{A t_c} = \frac{1}{a}\left[-\frac{R_m}{t_c} + \sqrt{\frac{R_m^2}{t_c^2} + \frac{2 f_k\ a\ X_s\ \Delta P}{m t_c}}\right]$$

can be used to describe the continuous operation of a rotary (vacuum) filter.

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Continuous Cake Filtration at Constant ΔP: Rotational Velocity

If the angular velocity is expressed in rpm, i.e.:

$$N = 60 \frac{\omega}{2\pi}$$

then the cycle time and the rotational (angular) velocities (in rad/s or rpm) are related by:

$$t_c = \frac{2\pi}{\omega} = \frac{60}{N}$$

where: ω = rotational (angular) velocity in rad/s

t_c = cycle time (to complete a rotation) in seconds

N = rotational (angular) velocity in rpm (rotations per minute).

Continuous Cake Filtration at Constant D*P*: Continuous Filtrate Flux

The continuous filtrate flux can be conveniently expressed in terms of the agitation velocity:

$$\text{Filtrate flux} = \frac{Q_F}{A} = \frac{1}{a\ X_s}\left[-\frac{R_m\, w}{2\, p} + \sqrt{\frac{R_m^2\, w^2}{(2\, p)^2} + \frac{f_k\, w\ a\ X_s\ \ \Delta P}{m\, p}}\right]$$

$$\text{Filtrate flux} = \frac{Q_F}{A} = \frac{1}{a\ X_s}\left[-\frac{R_m\, N}{60} + \sqrt{\frac{R_m^2\, N^2}{(60)^2} + \frac{2\, f_k\, N\, a\ X_s\ \ \Delta P}{60\, m}}\right]$$

where: t_c = cycle time (to complete a rotation), in s

N **= rotational (angular) velocity in rpm (rotations per minute).**

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Continuous Cake Filtration at Constant D*P*: Simplified Equations

If the resistance of the filter medium, R_m, is very small then the equations for continuous filtration can be simplified and rewritten as:

$$\text{Filtrate flux} = \frac{Q_F}{A} = \frac{V_F}{A t_c} \cong \sqrt{\frac{2 f_k \ \Delta P}{m a \ X_s \, t_c}}$$

$$G_k = \frac{V_F \, X_s}{A t_c} \cong \sqrt{\frac{2 f_k \, X_s \ \Delta P}{m a \, t_c}}$$

These equations are especially useful to understand the relationships between the various variables affecting a continuous filtration process.

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Continuous Cake Filtration at Constant D*P*: Simplified Equations

If the resistance of the filter medium, R_m, is very small the equations for continuous filtration can be expressed as a function of the rotational velocity as:

$$\text{Filtrate flux} = \frac{Q_F}{A} = \frac{V_F}{A t_c} \cong \sqrt{\frac{f_k \, \Delta P \, w}{p \, m \, a \, X_s}} \cong$$

$$\cong \sqrt{\frac{2 f_k \, \Delta P \, N}{60 \, m \, a \, X_s}}$$

where: t_c = cycle time (to complete a rotation) in s

N **= rotational (angular) velocity in rpm (rotations per minute).**

Applications of Continuous Cake Filtration at Constant ΔP

- **Continuous cake filtration at constant ΔP is the most widely used method of sludge dewatering;**
- **The types of filters using this filtration method include:**
  - **Rotary vacuum belt filter**
  - **Rotary vacuum precoat filters**
  - **Rotary vacuum drum filters**
  - **Rotary vacuum disc filters**

Comparison of Different Types of Filters

	Rotary Drum Belt Filters	Belt Filter Presses	Filter Presses	Granular Deep-Bed Filters
Size	1-70 m^2	1-2 m belt width	0.02-16 m^3	0.2-10 m^2
Solids in Feed (%)	2-5	2-8	--	--
Solids in Cake (%)	15-20	15-25	28-40	
Solids Loading	10 kg/m^2 h	190-270 kg/m h	--	0.12-0.5 m^3/m^2 min

After Freeman, *Standard Handbook of Hazardous Waste Treatment and Disposal*, 1989, p. 7.12

Design Information for Pressure Cake Filters

Cycle time: of the order of hours

Solids loading: 0.2-2 lb/ft·h

Solids in cake: up to 50%

Remark: although solids loading in pressure filters (e.g., filter presses) is typically smaller than that of vacuum filters the percentage of solids in the cake is typically higher. This is the result of the higher pressure that can be used in the operation of pressure filters (as opposed to a maximum of 1 atm in vacuum filters).

Design Information for Rotary Vacuum Filters

Diameter: up to 5 m

Length: up to 6 m

Vacuum levels: typically 20 in. Hg (68 kPa)

Submergence: 15-25% of drum area

	Primary Sludges	Waste-activated sludges
Solids loading	20-60 kg/m^2·h (4-12 lb/ft^2·h)	5-20 kg/m^2·h (1-4 lb/ft^2·h)
Solids in cake	25-40% (typically 20-25%)	10-15%

After Sundstrom and Klei, *Wastewater Treatment*, 1979, p. 234.

Operation of Rotary Vacuum Filters

- **Solids loading increases with increasing drum submergence, drum rotational speed, pressure difference across cake, solids concentration in feed.**
- **Percentage of solids in cake decreases with increasing drum submergence, and drum rotational speed.**
- **Rotary vacuum filters can be used to dewater sludges from activated sludge plants (biological sludges), chemical sludges, and sludges from precipitation operations.**

Additional Information and Examples on Cake Filtration

Additional information and examples on can be found in the following references:

- **Sundstrom, D. W. and Klei, H. E., 1979, *Wastewater Treatment*, Prentice Hall, Englewood Cliffs, NJ, p. 229-234.**
- **Geankoplis, C. J., *Transport Processes and Unit Operations*, 3rd Edition, 1993, Allyn and Bacon, Boston, pp. 800-815.**
- **Freeman, H. M. (ed.), 1989, *Standard Handbook of Hazardous Waste Treatment and Disposal*, McGraw-Hill, New York, pp. 7.3-7.19.**

Additional Information and Examples on Cake Filtration

- **Haas, C. N. and Vamos, R. J., 1995, *Hazardous and Industrial Waste Treatment*, Prentice Hall, Englewood Cliffs, NJ, pp. 75-78.**
- **Wentz, C. W., 1995, *Hazardous Waste Management*, Second Edition, McGraw-Hill, New York. pp. 196-200.**
- **Vesilind, P. A., 1979, *Treatment and Disposal of Wastewater Sludges,* Ann Arbor Science, Ann Arbor, MI, pp. 140-161.**